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State vector approach to analysis of multilayered magneto-electro-elastic plates

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Abstract

The state vector equations for three dimensional, orthotropic and linearly magneto-electro-elastic media are derived from the governing equations by eliminating σ_x, $\sigma_y \cdot \tau_{xy}$, B_x, B_y, D_x and D_y. An efficient method is presented for analysis of multilayered magneto-electro-elastic plates. The methodology is based on the mixed formulation, in which basic unknowns are formed by collecting not only displacements, electrical potential and magnetic potential but also some of stresses, electrical displacements, and magnetic induction. As special case, simply supported and multilayered rectangular plate is analyzed under the surface loading. Numerical results are presented graphically. The procedure of numerical calculation shows that the formulation presented here is simple and direct.

Keywords: Magneto-electro-elastic media; Multilayered plates; State vector approach; Exact solutions

1. Introduction

With the increasing technological applications of piezoelectric and piezomagnetic materials for development of "smart" or "intelligent" structures, the problem of magneto-electro-mechanical interaction in piezoelectric and piezomagnetic media has attracted considerable attention in the last few years. These materials exhibit magneto-electric-mechanical coupling effect in that they produce an electric field and a magnetic field when deformed and, conversely, undergo deformation when subjected to an electric field or a magnetic field. Some of the recent investigations have been devoted to magneto-electric effect of piezoelectric and piezomagnetic materials (Avellaneda and Harshe, 1994; Benveniste, 1995; Nan, 1994). Pan (2001) derived the exact solutions for three dimensional, anisotropic magneto-electro-elastic, simply supported, and multilayered rectangular plates by introducing the transfer matrix method.

The state vector method is an important method in analysis of multilayered structures (Bahar, 1972, 1975; Bufler, 1971; Das and Rao, 1977; Sundara Raja Iyengar and Pandya, 1983; Sosa and Castro, 1993; Benitez and Rosakis, 1987; Lee and Jiang, 1996; Wang, 1999a; Wang and Fang, 1999b; Wang, 2001a; Wang and

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Fang, 2001b; Wang et al., 2002). Based on the mixed formulation of solid mechanics, the state vector method converts a boundary value problem to an equivalent initial value problem. Once the transfer matrix of a single layer is obtained, a global matrix can be assembled by introducing interface continuity conditions. The order of the global matrix does not depend on the number of layers since the matrix is obtained by the multiplication of the transfer matrix of each single layer for certain interface continuity condition.

In this paper, state variable equations for the linear theory of magneto-electro-elastic materials are presented with the aim of deriving an efficient analytical method for multilayered magneto-electro-elastic structures. Using the state vector method developed in the study, an analytical solution is obtained for magneto-electro-elastic, simply supported and multilayered rectangular plates in the form of infinite series. The accuracy of the solutions can be controlled to any desired level by retaining an appropriate number of series terms. Some of numerical results are given.

2. Basic equations

In this section, we present a brief summary of basic equations for multilayered magneto-electro-elastic media in Cartesian coordinates. If body forces, electric charge density and magnetic charge density are ignored, the magneto-electro-elastic field for static cases is governed by

$$\begin{aligned}
&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}=0\\
&\frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}=0\\
&\frac{\partial\tau_{zx}}{\partial x}+\frac{\partial\tau_{zy}}{\partial y}+\frac{\partial\sigma_z}{\partial z}=0
\end{aligned}\tag{1}$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=0\tag{2}$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0\tag{3}$$

where σ_{ij}, D_j, and B_j are the stress components, electric displacement components, and magnetic induction components, respectively. For an orthotropic magneto-electro-elastic solid, with transversely isotropy being a special case, the coupled constitutive equation can be written in the following form:

$$\begin{aligned}
&\sigma_x=c_{11}\frac{\partial u}{\partial x}+c_{12}\frac{\partial v}{\partial y}+c_{13}\frac{\partial w}{\partial z}+e_{31}\frac{\partial\phi}{\partial z}+q_{31}\frac{\partial\psi}{\partial z}\\
&\sigma_y=c_{12}\frac{\partial u}{\partial x}+c_{22}\frac{\partial v}{\partial y}+c_{23}\frac{\partial w}{\partial z}+e_{32}\frac{\partial\phi}{\partial z}+q_{32}\frac{\partial\psi}{\partial z}\\
&\sigma_z=c_{13}\frac{\partial u}{\partial x}+c_{23}\frac{\partial v}{\partial y}+c_{33}\frac{\partial w}{\partial z}+e_{33}\frac{\partial\phi}{\partial z}+q_{33}\frac{\partial\psi}{\partial z}\\
&\tau_{zy}=c_{44}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)+e_{24}\frac{\partial\phi}{\partial y}+q_{24}\frac{\partial\psi}{\partial y}\\
&\tau_{zx}=c_{55}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+e_{15}\frac{\partial\phi}{\partial y}+q_{15}\frac{\partial\psi}{\partial y}\\
&\tau_{xy}=c_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)
\end{aligned}\tag{4}$$

$$D_x = e_{15}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial x} - d_{11}\frac{\partial \psi}{\partial x}$$

$$D_y = e_{24}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) - \varepsilon_{22}\frac{\partial \phi}{\partial y} - d_{22}\frac{\partial \psi}{\partial y} \tag{5}$$

$$D_z = e_{31}\frac{\partial u}{\partial x} + e_{32}\frac{\partial v}{\partial y} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \phi}{\partial z} - d_{33}\frac{\partial \psi}{\partial z}$$

$$B_x = q_{15}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) - d_{11}\frac{\partial \phi}{\partial x} - \mu_{11}\frac{\partial \psi}{\partial x}$$

$$B_y = q_{24}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) - d_{22}\frac{\partial \phi}{\partial y} - \mu_{22}\frac{\partial \psi}{\partial y} \tag{6}$$

$$B_z = q_{31}\frac{\partial u}{\partial x} + q_{32}\frac{\partial v}{\partial y} + q_{33}\frac{\partial w}{\partial z} - d_{33}\frac{\partial \phi}{\partial z} - \mu_{33}\frac{\partial \psi}{\partial z}$$

where c_{ij}, ε_{ij} and μ_{ij} are the elastic, dielectric, and magnetic permeability coefficients, respectively; e_{ij}, q_{ij} and d_{ij} are the piezoelectric, piezomagnetic, and magnetoelectric coefficients, respectively; u, v and w are the component of displacements; ϕ and ψ are the electric potential and magnetic potential, respectively. Eqs. (4)–(6) represent magneto-electro-elastic coupled constitutive relations. Various uncoupled cases can be reduced from Eqs. (4)–(6) by setting the appropriate coefficients to zero.

3. State vector formulation

The state vector approach is based on the mixed formulation of solid mechanics in which u, v, w, σ_z, D_z, B_z, τ_{zx}, τ_{zy}, ϕ and ψ are taken as basic unknowns. Following the process of state vector approach in piezoelasticity (Sosa and Castro, 1993; Lee and Jiang, 1996; Wang, 1999a, 2001a) and eliminating σ_x, σ_y, τ_{xy}, D_x, D_y, B_x and B_y from the governing equations (1)–(6), the field equations can be recast in the following matrix form:

$$\frac{\partial \eta_1}{\partial z} = [A]\eta_1 \tag{7a}$$

$$\eta_2 = [B]\eta_1 \tag{7b}$$

where η_1 is the basic unknown vector, which is called the state vector. η_2 is related to η_1 by Eq. (7b).

$$\eta_1 = [u \quad v \quad D_z \quad B_z \quad \sigma_z \quad \tau_{zx} \quad \tau_{zy} \quad \phi \quad \psi \quad w]^T \tag{8}$$

$$\eta_2 = [\sigma_x \quad \sigma_y \quad \tau_{xy} \quad D_x \quad D_y \quad B_x \quad B_y]^T \tag{9}$$

$$[A] = \begin{bmatrix} 0 & A_1 \\ A_2 & 0 \end{bmatrix}, \quad [B] = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \tag{10}$$

$$[A_1] = \begin{bmatrix} a & 0 & -\beta_1 \frac{\partial}{\partial x} & -\gamma_1 \frac{\partial}{\partial x} & -\frac{\partial}{\partial x} \\ 0 & b & -\beta_4 \frac{\partial}{\partial y} & -\gamma_3 \frac{\partial}{\partial y} & -\frac{\partial}{\partial y} \\ -\beta_1 \frac{\partial}{\partial x} & -\beta_4 \frac{\partial}{\partial y} & \beta_2 \frac{\partial^2}{\partial x^2} + \beta_5 \frac{\partial^2}{\partial y^2} & \beta_3 \frac{\partial^2}{\partial x^2} + \beta_6 \frac{\partial^2}{\partial y^2} & 0 \\ -\gamma_1 \frac{\partial}{\partial x} & -\gamma_3 \frac{\partial}{\partial y} & \beta_3 \frac{\partial^2}{\partial x^2} + \beta_6 \frac{\partial^2}{\partial y^2} & \gamma_2 \frac{\partial^2}{\partial x^2} + \gamma_4 \frac{\partial^2}{\partial y^2} & 0 \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$[A_2] = \begin{bmatrix} -\left(\alpha_1 \frac{\partial^2}{\partial x^2} + c_{66} \frac{\partial^2}{\partial y^2}\right) & -\left(\alpha_2 \frac{\partial^2}{\partial x \partial y} + c_{66} \frac{\partial^2}{\partial x \partial y}\right) & -\alpha_4 \frac{\partial}{\partial x} & -\alpha_5 \frac{\partial}{\partial x} & -\alpha_3 \frac{\partial}{\partial x} \\ -\left(\alpha_6 \frac{\partial^2}{\partial x \partial y} + c_{66} \frac{\partial^2}{\partial x \partial y}\right) & -\left(\alpha_7 \frac{\partial^2}{\partial y^2} + c_{66} \frac{\partial^2}{\partial x^2}\right) & -\alpha_9 \frac{\partial}{\partial x} & -\alpha_{10} \frac{\partial}{\partial x} & -\alpha_8 \frac{\partial}{\partial x} \\ -b_{21} \frac{\partial}{\partial x} & -b_{22} \frac{\partial}{\partial y} & a_4 & a_5 & a_2 \\ -b_{31} \frac{\partial}{\partial x} & -b_{32} \frac{\partial}{\partial y} & a_5 & a_6 & a_3 \\ -b_{11} \frac{\partial}{\partial x} & -b_{12} \frac{\partial}{\partial y} & a_2 & a_3 & a_1 \end{bmatrix} \tag{12}$$

$$[B_1] = \begin{bmatrix} \alpha_1 \frac{\partial}{\partial x} & \alpha_2 \frac{\partial}{\partial y} & \alpha_4 & \alpha_5 & \alpha_3 \\ \alpha_6 \frac{\partial}{\partial x} & \alpha_7 \frac{\partial}{\partial y} & \alpha_9 & \alpha_{10} & \alpha_8 \\ c_{66} \frac{\partial}{\partial y} & c_{66} \frac{\partial}{\partial x} & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$[B_2] = \begin{bmatrix} \beta_1 & 0 & -\beta_2 \frac{\partial}{\partial x} & -\beta_3 \frac{\partial}{\partial x} & 0 \\ 0 & \beta_4 & -\beta_5 \frac{\partial}{\partial y} & -\beta_6 \frac{\partial}{\partial y} & 0 \\ \gamma_1 & 0 & -\beta_3 \frac{\partial}{\partial x} & -\gamma_2 \frac{\partial}{\partial x} & 0 \\ 0 & \gamma_3 & -\beta_6 \frac{\partial}{\partial y} & -\gamma_4 \frac{\partial}{\partial y} & 0 \end{bmatrix} \tag{14}$$

$$\kappa a_1 = \varepsilon_{33}\mu_{33} - d_{33}^2, \quad \kappa a_2 = e_{33}\mu_{33} - d_{33}q_{33}, \quad \kappa a_3 = q_{33}\varepsilon_{33} - e_{33}d_{33}$$

$$\kappa a_4 = -c_{33}\mu_{33} - q_{33}^2, \quad \kappa a_5 = c_{33}d_{33} + e_{33}q_{33}, \quad \kappa a_6 = -c_{33}\varepsilon_{33} - e_{33}^2$$

$$\kappa = \varepsilon_{33}(c_{33}\mu_{33} + q_{33}^2) - d_{33}(c_{33}d_{33} + 2e_{33}q_{33}) + e_{33}^2\mu_{33}$$

$$b_{11} = a_1c_{13} + a_2e_{31} + a_3q_{31}, \quad b_{12} = a_1c_{23} + a_2e_{32} + a_3q_{32}, \quad b_{21} = a_2c_{13} + a_4e_{31} + a_5q_{31}$$

$$b_{22} = a_2c_{23} + a_4e_{32} + a_5q_{32}, \quad b_{31} = a_3c_{13} + a_5e_{31} + a_6q_{31}, \quad b_{32} = a_3c_{23} + a_5e_{32} + a_6q_{32}$$

$$\alpha_1 = c_{11} - c_{13}b_{11} - e_{31}b_{21} - q_{31}b_{31}, \quad \alpha_2 = c_{12} - c_{13}b_{12} - e_{31}b_{22} - q_{31}b_{32}$$

$$\alpha_3 = c_{13}a_1 + e_{31}a_2 + q_{31}a_3, \quad \alpha_4 = c_{13}a_2 + e_{31}a_4 + q_{31}a_5, \quad \alpha_5 = c_{13}a_3 + e_{31}a_5 + q_{31}a_6$$
$$\alpha_6 = c_{12} - c_{23}b_{11} - e_{32}b_{21} - q_{32}b_{31}, \quad \alpha_7 = c_{22} - c_{23}b_{12} - e_{32}b_{22} - q_{32}b_{32}$$
$$\alpha_8 = c_{23}a_1 + e_{32}a_2 + q_{32}a_3, \quad \alpha_9 = c_{23}a_2 + e_{32}a_4 + q_{32}a_5, \quad \alpha_{10} = c_{23}a_3 + e_{32}a_5 + q_{32}a_6$$
$$a = \frac{1}{c_{55}}, \quad b = \frac{1}{c_{44}}, \quad \beta_1 = ae_{15}, \quad \beta_2 = ae_{15}^2 + \varepsilon_{11}, \quad \beta_3 = aq_{15}e_{15} + d_{11}, \quad \beta_4 = be_{24}$$
$$\beta_5 = be_{24}^2 + \varepsilon_{22}, \quad \beta_6 = bq_{24}e_{24} + d_{22}, \quad \gamma_1 = aq_{15}, \quad \gamma_2 = aq_{15}^2 + \mu_{11}, \quad \gamma_3 = bq_{24}$$
$$\gamma_4 = bq_{24}^2 + \mu_{22} \tag{15}$$

Eq. (7a) is a system of partial differential equations taking state vector as basic unknowns. If the solution of equation (7a) is obtained, we can obtain the solution of Eq. (7b) by differential procedure.

4. Analytical solutions of simply supported plates

In this section, we employ the state vector equation presented in the above section to find an analytical solution for simply supported on all four edges and multilayered magneto-electro-elastic rectangular plates. We assume that a Cartesian coordinate system (x, y, z) is attached to the plate. The origin of the coordinate system is located at one of the four corners on the bottom surface, and the z-axis is normal to the plate. Let z_j denote the coordinate of the lower interface of the jth layer and z_{j+1} the coordinate of the upper interface of the jth layer. The thickness of the jth layer is $h_j = z_{j+1} - z_j$. Material properties in each layer can be different, and surface loads (mechanical, electric or magnetic) can be applied. Along the interface, the displacement and traction vectors are assumed to be continuous except the internal loading is applied. For simplicity, we assume that the surface load is applied on the top surface of the layered plates. We assume that N-layered rectangular plates have dimensions L_x in x-direction and L_y in y-direction and total thickness H (in the vertical direction). u denotes the displacement of x-direction, v the displacement of y-direction, and w the displacement of z-direction. The boundary conditions of state vector can be written as follows:

$$v = w = \phi = \psi = D_z = B_z = \sigma_z = \tau_{zy} = 0 \quad \text{at } x = 0, \quad \text{and} \quad x = L_x$$
$$u = w = \phi = \psi = D_z = B_z = \sigma_z = \tau_{zx} = 0 \quad \text{at } y = 0, \quad \text{and} \quad y = L_y \tag{16}$$

The state variables, which exactly satisfy the boundary conditions, can be expressed in the following form:

$$\begin{pmatrix} u \\ v \\ D_z \\ B_z \\ \sigma_z \\ \tau_{zx} \\ \tau_{zy} \\ \phi \\ \varphi \\ w \end{pmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{pmatrix} \bar{u}_{mn}(z)\cos\xi x \sin\eta y \\ \bar{v}_{mn}(z)\sin\xi x\cos\eta y \\ \overline{D}_{zmn}(z)\sin\xi x\sin\eta y \\ \overline{B}_{zmn}(z)\sin\xi x\sin\eta y \\ \bar{\sigma}_{zmn}(z)\sin\xi x\sin\eta y \\ \bar{\tau}_{zxmn}(z)\cos\xi x\sin\eta y \\ \bar{\tau}_{zymn}(z)\sin\xi x\cos\eta y \\ \bar{\phi}_{mn}(z)\sin\xi x\sin\eta y \\ \overline{\psi}_{mn}(z)\sin\xi x\sin\eta y \\ \bar{w}_{mn}(z)\sin\xi x\sin\eta y \end{pmatrix} \tag{17}$$

where $\xi = n\pi/L_x$ and $\eta = m\pi/L_y$. Substituting Eq. (17) into Eqs. (7a) and (7b), we obtain a system of ordinary differential equations, which are expressed in the following matrix form.

$$\frac{d\bar{\eta}_1}{dz} = [\bar{A}]\bar{\eta}_1 \tag{18a}$$

$$\bar{\eta}_2 = [\bar{B}]\bar{\eta}_1 \tag{18b}$$

where

$$\bar{\eta}_{1mn} = \begin{bmatrix} \bar{u}_{mn} & \bar{v}_{mn} & \overline{D}_{zmn} & \overline{B}_{zmn} & \bar{\sigma}_{zmn} & \bar{\tau}_{zxmn} & \bar{\tau}_{zymn} & \bar{\phi}_{mn} & \overline{\psi}_{mn} & \bar{w}_{mn} \end{bmatrix}^T \tag{19}$$

$$\bar{\eta}_{2mn} = \begin{bmatrix} \bar{\sigma}_{xmn} & \bar{\sigma}_{ymn} & \bar{\tau}_{xymn} & \overline{D}_{xmn} & \overline{D}_{ymn} & \overline{B}_{xmn} & \overline{B}_{ymn} \end{bmatrix}^T \tag{20}$$

$$[\bar{A}] = \begin{bmatrix} 0 & \bar{A}_1 \\ \bar{A}_2 & 0 \end{bmatrix} \quad [\bar{B}] = \begin{bmatrix} \bar{B}_1 & 0 \\ 0 & \bar{B}_2 \end{bmatrix} \tag{21}$$

$$[\bar{A}_1] = \begin{bmatrix} a & 0 & -\beta_1\xi & -\gamma_1\xi & -\xi \\ 0 & b & -\beta_4\eta & -\gamma_3\eta & -\eta \\ \beta_1\xi & \beta_4\eta & -(\beta_2\xi^2+\beta_5\eta^2) & -(\beta_2\xi^2+\beta_6\eta^2) & 0 \\ \gamma_1\xi & \gamma_3\eta & -(\beta_3\xi^2+\beta_6\eta^2) & -(\gamma_2\xi^2+\gamma_4\eta^2) & 0 \\ \xi & \eta & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$[\bar{A}_2] = \begin{bmatrix} (\alpha_1\xi^2+c_{66}\eta^2) & (\alpha_2+c_{66})\xi\eta & -\alpha_4\xi & -\alpha_5\xi & -\alpha_3\xi \\ (\alpha_6+c_{66})\xi\eta & (\alpha_7\xi^2+c_{66}\eta^2) & -\alpha_9\eta & -\alpha_{10}\eta & -\alpha_8\eta \\ b_{21}\xi & b_{22}\eta & a_4 & a_5 & a_2 \\ b_{31}\xi & b_{32}\eta & a_5 & a_6 & a_3 \\ b_{11}\xi & b_{12}\eta & a_2 & a_3 & a_1 \end{bmatrix} \tag{23}$$

$$[\bar{B}_1] = \begin{bmatrix} -\alpha_1\xi & -\alpha_2\eta & \alpha_4 & \alpha_5 & \alpha_3 \\ -\alpha_6\xi & -\alpha_7\eta & \alpha_9 & \alpha_{10} & \alpha_8 \\ c_{66}\eta & c_{66}\xi & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

$$[\bar{B}_2] = \begin{bmatrix} \beta_1 & 0 & -\beta_2\xi & -\beta_3\xi & 0 \\ 0 & \beta_4 & -\beta_5\eta & -\beta_6\eta & 0 \\ \gamma_1 & 0 & -\beta_3\xi & -\gamma_2\xi & 0 \\ 0 & \gamma_3 & -\beta_6\eta & -\gamma_4\eta & 0 \end{bmatrix} \tag{25}$$

According to the theory of the solution of ordinary differential equations (Bellman, 1970), the solutions of state vector equations can be expressed as follows.

$$\bar{\eta}_{1mn}(z) = \exp(Az)\bar{\eta}_{1mn}(0) = [G]\bar{\eta}_{1mn}(0) \tag{26}$$

where $[G] = \exp(Az)$, $\exp(Az)$ is a matrix exponential function. Complex algebraic manipulations involving the calculation of $\exp(Az)$ can be avoided because $\exp(Az)$ can be directly calculated by using bulit-in function in *Mathematica* or *Matlab*. For N-multilayered magneto-electro-elastic plates, there are

$$\begin{aligned} &\bar{\eta}_{1mn}(z_1) = [G(h_1)]\bar{\eta}_{1mn}(0) \\ &\bar{\eta}_{1mn}(z_2) = [G(h_2)]\bar{\eta}_{1mn}(z_1) \\ &\vdots \\ &\bar{\eta}_{1mn}(z_N) = [G(h_N)]\bar{\eta}_{1mn}(z_{N-1}) \end{aligned} \tag{27}$$

Considering the conditions of interface continuity, we have

$$\begin{aligned} &\bar{\eta}_{1mn}(z_N) = [T]\bar{\eta}_{1mn}(0) \\ &[T] = [G(h_N)]\cdots[G(h_2)][G(h_1)] \end{aligned} \tag{28}$$

For multilayered magneto-electro-elastic plates, if we assume that the stresses, electric displacements, and magnetic induction of the upper and bottom surface are known, their displacements, electric potential, and magnetic potential are unknown. Rearranging Eq. (28), we have

$$\left\{ \begin{matrix} \overline{U}_{mn}(z_N) \\ \overline{F}_{mn}(z_N) \end{matrix} \right\} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \left\{ \begin{matrix} \overline{U}_{mn}(0) \\ \overline{F}_{mn}(0) \end{matrix} \right\} \tag{29}$$

Solving the above equation, we obtain the following solutions:

$$\begin{aligned} &\{\overline{U}_{mn}(0)\} = [T_{21}]^{-1}(\{\overline{F}_{mn}(z_N)\} - [T_{22}]\{\overline{F}_{mn}(0)\}) \\ &\{\overline{U}_{mn}(z_N)\} = [T_{11}][T_{21}]^{-1}\{\overline{F}_{mn}(z_N)\} + ([T_{12}] - [T_{11}][T_{21}]^{-1}[T_{22}])\{\overline{F}_{mn}(0)\} \end{aligned} \tag{30}$$

where

$$\begin{aligned} &\{\overline{U}_{mn}\} = \begin{bmatrix} \bar{u}_{mn} & \bar{v}_{mn} & \bar{\phi}_{mn} & \overline{\psi}_{mn} & \bar{w}_{mn} \end{bmatrix}^T \\ &\{\overline{F}_{mn}\} = \begin{bmatrix} \overline{D}_{zmn} & \overline{B}_{zmn} & \bar{\sigma}_{zmn} & \bar{\tau}_{zxmn} & \bar{\tau}_{zymn} \end{bmatrix}^T \end{aligned} \tag{31}$$

In order to obtain the generalized displacements and tractions at any depth, say $z_k \leqslant z \leqslant z_{k+1}$ in layer k, we can evaluate the solutions using Eqs. (26) and (27). With the generalized displacement and traction vectors at a given depth being solved, the corresponding in-plane quantities can be evaluated using Eq. (18b).

If σ_z, D_z and B_z applied on the surface of the plate are known complex function about x and y, we can expand σ_z, D_z and B_z into infinite double Fourier series and then adding the responses together term by term.

$$\left\{ \begin{matrix} \bar{\sigma}_{zmn} \\ \overline{D}_{zmn} \\ \overline{B}_{zmn} \end{matrix} \right\} = \frac{4}{L_x L_y} \int_0^{L_x} \int_0^{L_y} \left\{ \begin{matrix} \sigma_z \\ D_z \\ B_z \end{matrix} \right\} \sin \xi x \sin \eta y \, \mathrm{d}x \, \mathrm{d}y \tag{32}$$

In the above equation, $\bar{\sigma}_{zmn}$, $\overline{D}_{zmn}$ and $\overline{B}_{zmn}$ are the coefficients of the Fourier series.

5. Numerical examples

In this section, we present some numerical results by using the formulation presented in this paper. Numerical calculation is completed by using *Mathematica 4.0*. The numerical example is for three-multi-layered plates made of piezoelectric $BaTiO_3$ and magnetostrictive $CoFe_2O_4$. Three layers have equal thickness of 0.1 m. The material properties for $BaTiO_3$ and $CoFe_2O_4$ are listed in Tables 1 and 2, respectively.

Table 1
Material coefficients of the piezoelectric $BaTiO_3$ (C_{ij} in 10^9 N/m^2, e_{ij} in C/m^2, ε_{ij} in 10^{-9} C^2/(N m^2), and μ_{ij} in 10^{-6} Ns^2/C^2)

$C_{11} = C_{22}$	C_{12}	$C_{13} = C_{23}$	C_{33}	$C_{44} = C_{55}$	C_{66}	
166	77	78	162	43	44.5	
$e_{31} = e_{32}$	e_{33}	$e_{24} = e_{15}$	$\varepsilon_{11} = \varepsilon_{22}$	ε_{33}	$\mu_{11} = \mu_{22}$	μ_{33}
−4.4	18.6	11.6	11.2	12.6	5	10

Table 2
Material coefficients of the magnetostrictive $CoFe_2O_4$ (C_{ij} in 10^9 N/m^2, q_{ij} in N/Am, ε_{ij} in 10^{-9} C^2/(N m^2), and μ_{ij} in 10^{-6} Ns^2/C^2)

$C_{11} = C_{22}$	C_{12}	$C_{13} = C_{23}$	C_{33}	$C_{44} = C_{55}$	C_{66}	
286	173	170.5	269.5	45.3	56.5	
$q_{31} = q_{32}$	q_{33}	$q_{24} = q_{15}$	$\varepsilon_{11} = \varepsilon_{22}$	ε_{33}	$\mu_{11} = \mu_{22}$	μ_{33}
580.3	699.7	550	0.08	0.093	−590	157

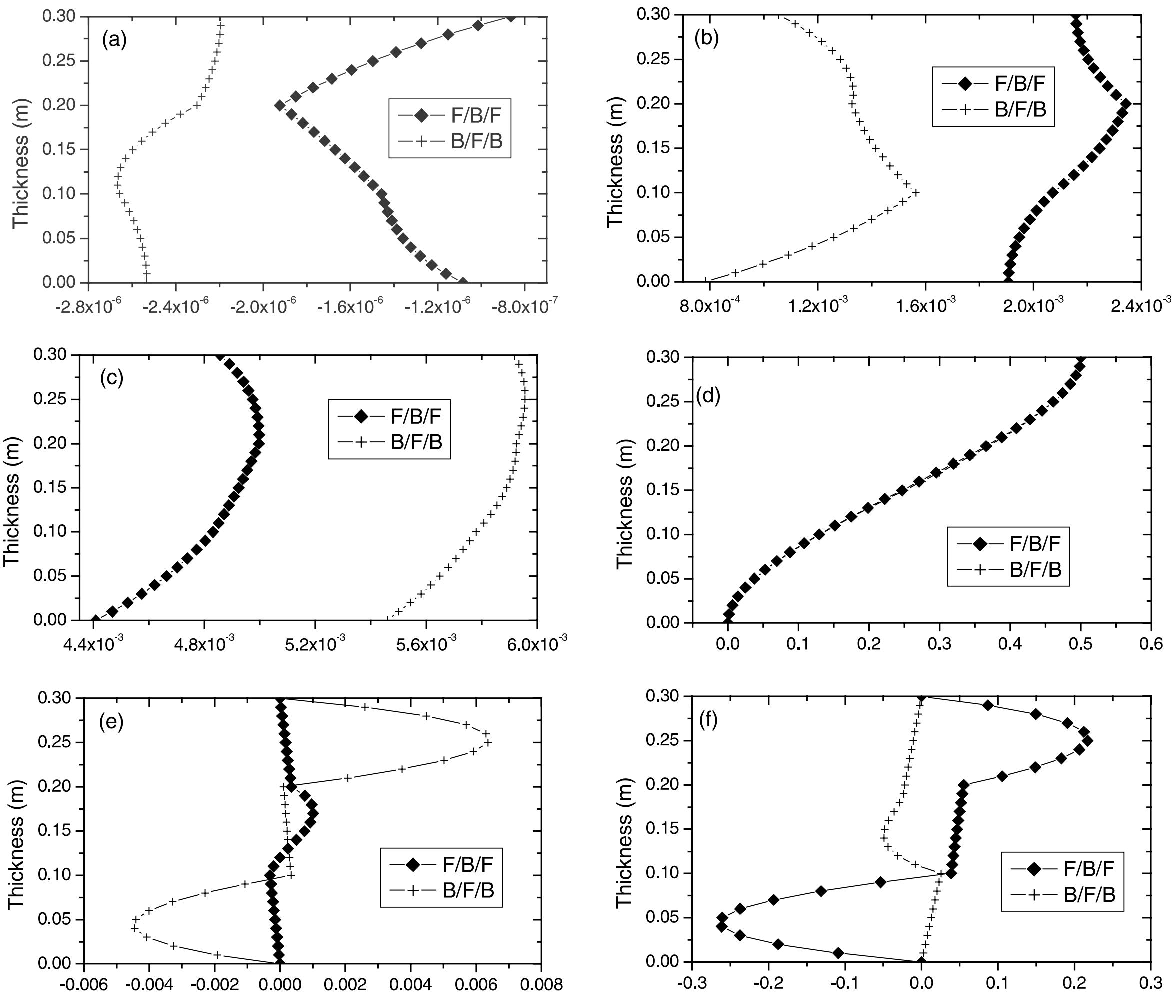

Fig. 1. (a) Magnetic potential ψ (C/s), (b) electric potential ϕ (V), (c) norma displacement $w \times 10^9$ (m), (d) Z-normal stress σ_Z (N/m^2), (e) Z-electric displacement D_Z (C/m^2), (f) Z-magnetic induction B_Z (Wb/m^2).

The piezoelectric $BaTiO_3$ and the magnetostrictive $CoFe_2O_4$ are homogeneous transversely isotropic solid with its symmetry axis along the z-axis. Two sandwich plates with stacking sequences $BaTiO_3$/$CoFe_2O_4$/$BaTiO_3$ (called B/F/B) and $CoFe_2O_4$/$BaTiO_3$/$CoFe_2O_4$ (called F/B/F) are investigated.

Machanical loading: A simply supported square lamina ($L_x = L_y = 1$ m) of thickness 0.3 m is subjected to a mechanical load on the top surface with the following sinusoidal distribution.

$$\sigma_z = \sigma_0 \sin(\pi x/L_x)\sin(\pi y/L_y) \tag{33}$$

where $\sigma_0 = 1$ N/m^2. Responses are calculated for fixed horizontal coordinates $(x, y) = (0.75L_x, 0.25L_y)$. The numerical results are presented in Fig. 1(a)–(f).

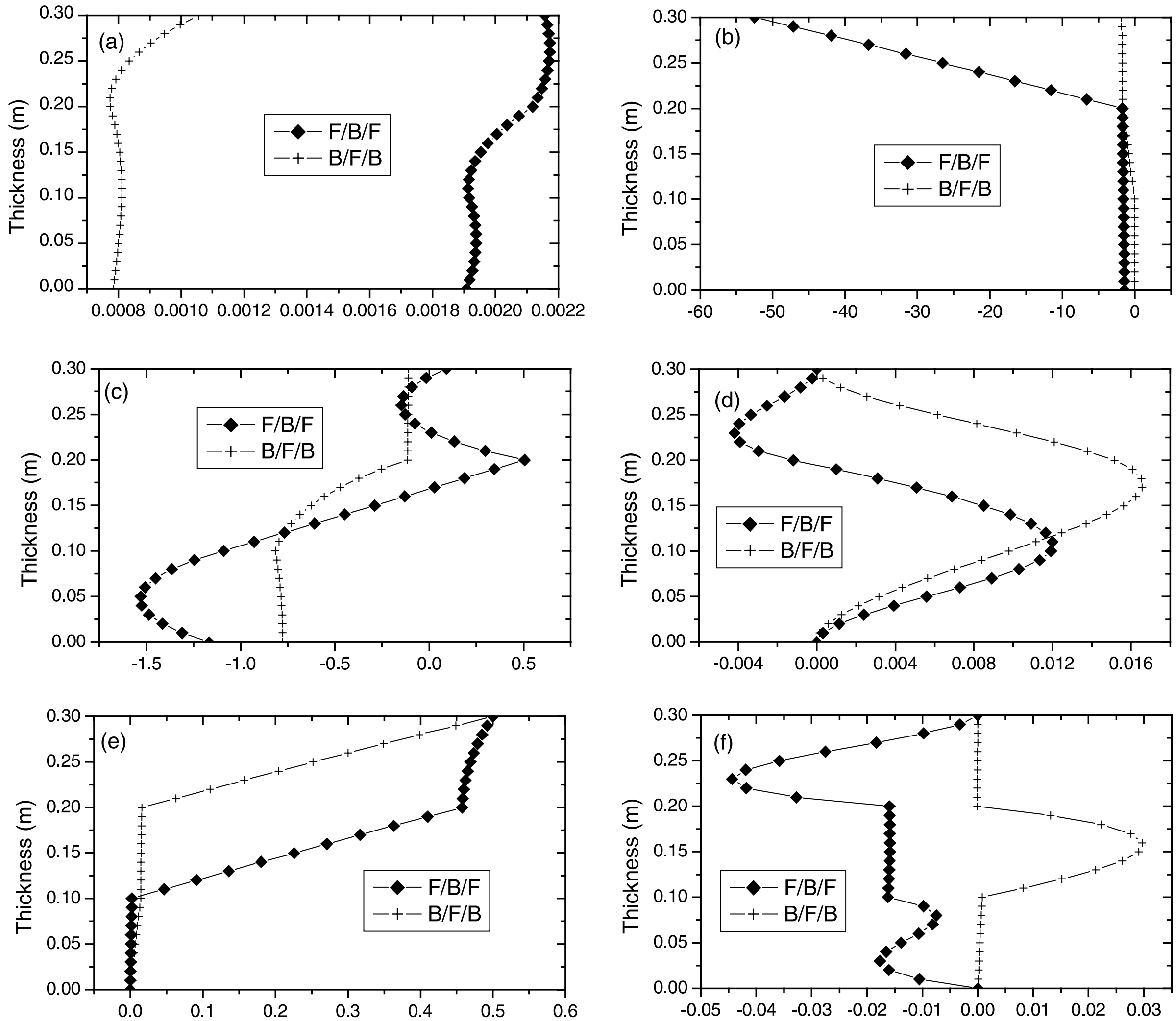

Fig. 2. (a) Normal displacement w (m), (b) electric potential ϕ 10^{-7} (V), (c) magnetic potential ψ 10^{-2} (C/s), (d) normal stress σ_Z 10^{-9} (N/m^2), (e) normal electric displacement D_Z (C/m^2), (f) normal magnetic induction B_Z (Wb/m^2), (g) shear stress τ_{zx} 10^{-9} (N/m^2), (h) electric displacement D_x (C/m^2), (i) magnetic induction B_x (Wb/m^2).

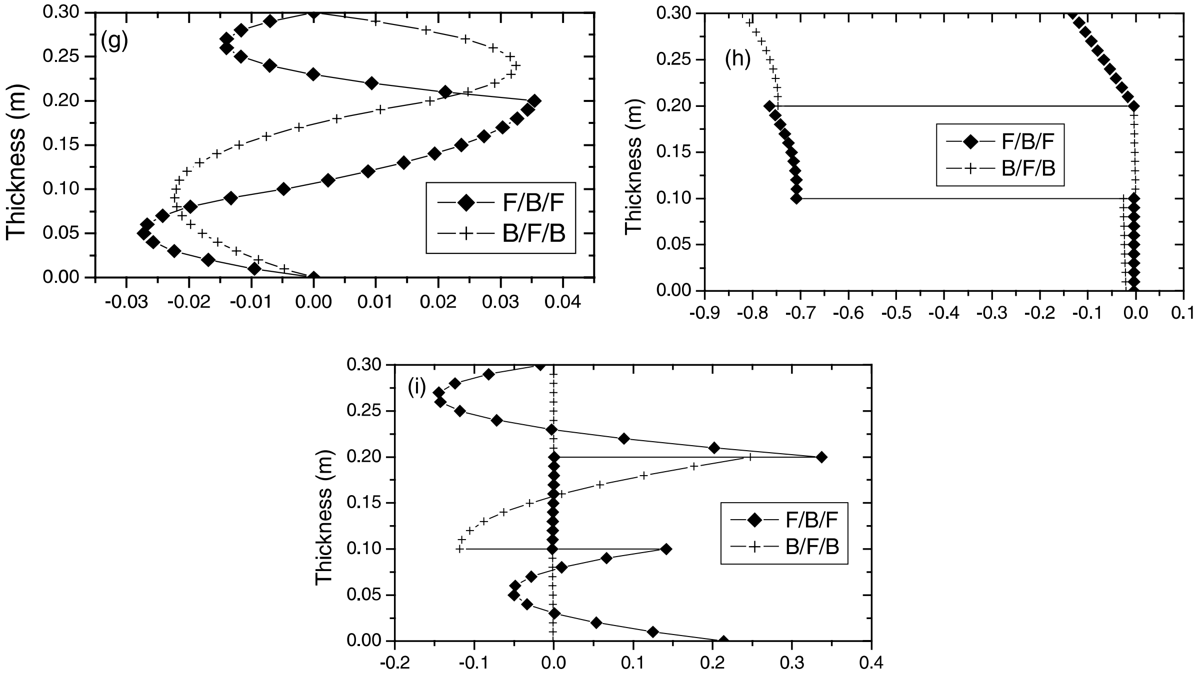

Fig. 2 (*continued*)

Electric loading: The same plate is loaded electrically on the top surface by transverse electric displacement of sinusoidal distribution

$$D_z = D_0 \sin(\pi x/L_x)\sin(\pi y/L_y) \tag{34}$$

where $D_0 = 1$ C/m^2. The responses are calculated at the same location as the previous mechanical loaded plates. The numerical results are given in Fig. 2(a)–(i).

For mechanical loading, we checked our solutions with the previously published results (Pan, 2001) and found that the present formulation agrees with these solutions. We also present some new results in Fig. 2(a)–(i) for electric loading.

Fig. 1 shows variation of ϕ, ψ, w, σ_z, D_z, B_z along the thickness direction in sandwich piezoelectric/piezomagnetic plate under mechanical loading on the top surface. Fig. 2 shows variation of ϕ, ψ, w, σ_z, D_z, B_z, τ_{zx}, D_x, B_x along the thickness direction in sandwich piezoelectric/piezomagnetic plate under electric loading on the top surface. Two sandwich plates with stacking sequences B/F/B and F/B/F, on which the mechanical and electrical loading are applied, are investigated. For the mechanical loading, the present solutions agree with the previously published results (Pan, 2001). For the electrical loading, the following general features are observed from Fig. 2(a)–(i). The elastic, electric and magnetic quantities have been greatly influenced by the stacking sequences. Fig. 2(d) and (g) shows that the stresses σ_z and τ_{zx} are completely different for the B/F/B and F/B/F stacking sequences. This phenomenon does not exist for the mechanical loading. Fig. 2(f) shows that the positive and negative of B_z along the thickness direction have been changed for the B/F/B and F/B/F stacking sequences. Fig. 2(h) and (i) shows that D_x and B_x along the thickness are discontinuous for B/F/B and F/B/F.

6. Conclusions

The state vector equations are presented for three dimensional, orthotropic and magneto-electro-elastic media. The method is based on the mixed formulation of solid mechanics, in which not only displacements, electric potential and magnetic potential but also some of stresses, electric displacements and magnetic induction are taken as basic unknowns. As special case, an exact solution is obtained for three dimensional, transversely isotropic, magneto-electro-elastic and multilayered rectangular plates with simply supported. For mechanical loading, numerical results presented here agree with those presented by Pan (2001). Some of new results are also presented for the electrical loading in which the elastic, electrical and magnetic quantities have been influenced by the stacking sequences. The advantage of the present method is that the order of global transfer matrix does not depend on the number of layers because all intermediate state vectors have been eliminated by using the continuity condition of the interface. Complicated algebraic manipulations involving the calculation of $\exp(zA)$ can be avoided because $\exp(zA)$ can be directly calculated by using built-in function in *Mathematica* or *Matlab*. The derived procedure and formulation presented in this paper is simple and elegant. The state vector approach developed herein can readily be extended for the study of transversely isotropic, magneto-Electro-elastic multilayered rectangular plates with general interlayer and boundary conditions. The coupling phenomenon of the magneto-electro-elastic field has potential applications in the field of smart structures. The problem of plates with more general boundary conditions at the edges is under study.

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